Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

8316475545

FURTHER MATHEMATICS **9231/13**

Paper 1 **October/November 2018**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **27** printed pages and **1** blank page.

JC18 11_9231_13/FP

CAMBRIDGE
International Examinations

1 The vectors **a**, **b**, **c** and **d** in $\mathbb{R}^3$ are given by

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 9 \\ 0 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{d} = \begin{pmatrix} 0 \\ -8 \\ 3 \end{pmatrix}.$$

(i) Show that {**a**, **b**, **c**} is a basis for $\mathbb{R}^3$. [3]

(ii) Express **d** in terms of **a**, **b** and **c**. [2]

2 The roots of the equation

$$x^3 + px^2 + qx + r = 0$$

are α, 2α, 4α, where p, q, r and α are non-zero real constants.

(i) Show that

$$2p\alpha + q = 0.$$ [4]

(ii) Show that

$$p^3 r - q^3 = 0.$$ [2]

3 The sequence of positive numbers $u_1, u_2, u_3, \ldots$ is such that $u_1 < 3$ and, for $n \geqslant 1$,

$$u_{n+1} = \frac{4u_n + 9}{u_n + 4}.$$

(i) By considering $3 - u_{n+1}$, or otherwise, prove by mathematical induction that $u_n < 3$ for all positive integers n. [5]

(ii) Show that $u_{n+1} > u_n$ for $n \geqslant 1$. [3]

4 A curve is defined parametrically by

$$x = t - \tfrac{1}{2}\sin 2t \quad \text{and} \quad y = \sin^2 t.$$

The arc of the curve joining the point where $t = 0$ to the point where $t = \pi$ is rotated through one complete revolution about the x-axis. The area of the surface generated is denoted by S.

(i) Show that

$$S = a\pi \int_0^{\pi} \sin^3 t \, \mathrm{d}t,$$

where the constant a is to be found. [5]

(ii) Using the result $\sin 3t = 3\sin t - 4\sin^3 t$, find the exact value of S. [3]

[Turn over

5 It is given that λ is an eigenvalue of the matrix $\mathbf{A}$ with $\mathbf{e}$ as a corresponding eigenvector, and μ is an eigenvalue of the matrix $\mathbf{B}$ for which $\mathbf{e}$ is also a corresponding eigenvector.

(i) Show that $\lambda + \mu$ is an eigenvalue of the matrix $\mathbf{A} + \mathbf{B}$ with $\mathbf{e}$ as a corresponding eigenvector. [2]

....

....

....

....

....

....

....

....

The matrix $\mathbf{A}$, given by

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{pmatrix}$$

has $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ as eigenvectors.

(ii) Find the corresponding eigenvalues. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

The matrix **B** has eigenvalues 4, 5 and 1 with corresponding eigenvectors $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ respectively.

(iii) Find a matrix **P** and a diagonal matrix **D** such that $(\mathbf{A} + \mathbf{B})^3 = \mathbf{PDP}^{-1}$. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

6 The curve C has equation

$$y = \frac{x^2 + ax - 1}{x + 1},$$

where a is constant and $a > 1$.

(i) Find the equations of the asymptotes of C. [3]

(ii) Show that C intersects the x-axis twice. [1]

(iii) Justifying your answer, find the number of stationary points on C. [2]

....

....

....

....

....

....

....

....

....

....

....

....

(iv) Sketch C, stating the coordinates of its point of intersection with the y-axis. [3]

7 **(i)** Use de Moivre's theorem to show that

$$\sin 8\theta = 8\sin\theta\cos\theta(1 - 10\sin^2\theta + 24\sin^4\theta - 16\sin^6\theta).$$ [6]

....

....

....

....

....

....

....

(ii) Use the equation $\frac{\sin 8\theta}{\sin 2\theta} = 0$ to find the roots of

$$16x^6 - 24x^4 + 10x^2 - 1 = 0$$

in the form $\sin k\pi$, where k is rational. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

8 The plane Π_1 has equation

$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix} + s\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

(i) Find a cartesian equation of Π_1. [3]

....

....

....

....

....

....

....

....

....

....

....

The plane Π_2 has equation $3x + y - z = 3$.

(ii) Find the acute angle between Π_1 and Π_2, giving your answer in degrees. [2]

....

....

....

....

....

....

....

....

....

(iii) Find an equation of the line of intersection of Π_1 and Π_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$. [5]

9 The curve C has polar equation

$$r = 5\sqrt{(\cot\theta)},$$

where $0.01 \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Find the area of the finite region bounded by C and the line $\theta = 0.01$, showing full working. Give your answer correct to 1 decimal place. [3]

Let P be the point on C where $\theta = 0.01$.

(ii) Find the distance of P from the initial line, giving your answer correct to 1 decimal place. [2]

(iii) Find the maximum distance of C from the initial line. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(iv) Sketch C. [2]

10 **(i)** Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 37\sin 3t,$$

given that $x = 3$ and $\frac{dx}{dt} = 0$ when $t = 0$. [10]

(ii) Show that, for large positive values of t and for any initial conditions,

$$x \approx \sqrt{(37)}\,\sin(3t - \phi),$$

where the constant ϕ is such that $\tan\phi = 6$. [3]

11 Answer only **one** of the following two alternatives.

EITHER

(i) By considering $(2r+1)^2 - (2r-1)^2$, use the method of differences to prove that

$$\sum_{r=1}^{n} r = \tfrac{1}{2}n(n+1).$$ [3]

(ii) By considering $(2r + 1)^4 - (2r - 1)^4$, use the method of differences and the result given in part **(i)** to prove that

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n + 1)^2. \qquad [5]$$

The sums S and T are defined as follows:

$$S = 1^3 + 2^3 + 3^3 + 4^3 + \ldots + (2N)^3 + (2N+1)^3,$$
$$T = 1^3 + 3^3 + 5^3 + 7^3 + \ldots + (2N-1)^3 + (2N+1)^3.$$

(iii) Use the result given in part **(ii)** to show that $S = (2N+1)^2(N+1)^2$. [1]

....

....

....

....

(iv) Hence, or otherwise, find an expression in terms of N for T, factorising your answer as far as possible. [2]

....

....

....

....

....

....

....

....

....

....

....

(v) Deduce the value of $\dfrac{S}{T}$ as $N \to \infty$. [2]

....

....

....

....

OR

The curve C has equation

$$x^2 + 2xy = y^3 - 2.$$

(i) Show that $A\,(-1,\ 1)$ is the only point on C with x-coordinate equal to -1. [2]

For $n \geqslant 1$, let A_n denote the value of $\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$ at the point $A\,(-1,\ 1)$.

(ii) Show that $A_1 = 0$. [3]

(iii) Show that $A_2 = \frac{2}{5}$. [3]

Let $I_n = \int_{-1}^{0} x^n \frac{\mathrm{d}^n y}{\mathrm{d}x^n}\,\mathrm{d}x$.

(iv) Show that for $n \geqslant 2$,

$$I_n = (-1)^{n+1}A_{n-1} - nI_{n-1}. \qquad [3]$$

(v) Deduce the value of I_3 in terms of I_1. [2]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE